# Barème CC1 Stat Inférentielle 20/21

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**Ex1)** $(X_n)$ iid , $X_i \rightsquigarrow \mathcal{P}(1)$ , $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ , $S_n = \sum_{i=1}^{n} X_i$

3) 1) Selon TCL $\bar{X}_n \underset{n}{\rightsquigarrow} \mathcal{N}(E(X_i), V(X_i)/n)$ 1

Or $\overset{0.5}{E(X_i) = 1}$ et $\overset{0.5}{V(X_i) = 1}$ alors $\bar{X}_n \underset{n}{\rightsquigarrow} \mathcal{N}(1, 1/n)$ 1

2) 2) $S_n = n\bar{X}_n$ 1 alors $E(S_n) = nE(\bar{X}_n) = n$

et $V(S_n) = n^2 V(\bar{X}_n) = n^2 \times 1/n = n$

$\Rightarrow S_n \underset{n}{\rightsquigarrow} \mathcal{N}(n, n)$ 1

2) 3) • $S_n \underset{n}{\rightsquigarrow} \mathcal{N}(n,n)$ alors $Z = \frac{S_n - n}{\sqrt{n}} \underset{n}{\rightsquigarrow} \mathcal{N}(0,1)$ 1

$\lim_{n\to\infty} P(S_n \le n) = \lim_{n\to\infty} P(S_n - n \le 0) = \lim_{n\to\infty} P(Z \le 0) = 1/2.$ 1

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**Ex2)** 1) a) $E(X) = \lambda$ alors $\bar{X}_n = \lambda$ , $\bar{X}_n$ est l'EMM de $\lambda$. 1

b) • $L(\lambda) = \prod_{i=1}^{n} P(X = x_i) = \prod_{i=1}^{n} e^{-\lambda} \frac{\lambda^{x_i}}{x_i!}$ 1

4) • $\ell(\lambda) = \log L(\lambda) = \sum_{i=1}^{n}\left[\ln(e^{-\lambda}) + \ln \frac{\lambda^{x_i}}{x_i!}\right]$

$= \sum_{i=1}^{n}[-\lambda + x_i \ln\lambda - \ln x_i!]$ 1

• $\frac{d\ell(\lambda)}{d\lambda} = \sum_{i=1}^{n}[-1 + x_i/\lambda] = 0$ alors $-n + \frac{\sum x_i}{\lambda} = 0$ 1

$\Rightarrow \frac{\sum x_i}{\lambda} = n \Rightarrow \lambda = \frac{1}{n}\sum x_i = \bar{X}_n.$

$\Rightarrow \bar{X}_n$ est l'EMV de $\lambda$.

2) Soit $f(x) = (\theta + 1)x^{\theta} \times \mathbb{1}_{]0,1[}(x)$

3) * $L(\theta) = \prod_{i=1}^{n} (\theta+1) x_i^{\theta}$ 0.5

* $\ell(\theta) = \sum_{i=1}^{n}[\ln(\theta+1) + \theta\ln(x_i)]$ 1

* $\frac{d\ell(\theta)}{d\theta} = \sum_{i=1}^{n}\left[\frac{1}{1+\theta} + \ln x_i\right] = 0$ 0.5 alors $\frac{n}{1+\theta} = -\sum_{i=1}^{n} \ln x_i$

et $1 + \theta = -\frac{n}{\sum \ln x_i}$ $\Rightarrow \theta = 1 + \frac{n}{\sum_{i=1}^{n} \ln x_i}$ 1 l'EMV de $\theta$.

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**Ex3**

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1) * $E(\theta_1) = E\left(\frac{X_1+X_2}{2}\right) = \frac{1}{2}(\mu+\mu) = \mu \Rightarrow$ non biaisé 1

* $E(\theta_2) = E\left(\frac{X_1+3X_2}{4}\right) = \frac{1}{4}\mu + \frac{3}{4}\mu = \mu \Rightarrow$ non biaisé 1

2) Non biaisés alors on compare leurs variances. 1

car $X_1$ et $X_2$ sont indép.

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* $V(\theta_1) = 1/4\left[V(X_1)+V(X_2)\right] = \sigma^2/2$. 1

* $V(\theta_2) = 1/16\, V(X_1) + 9/16\, V(X_2) = 10/16\, \sigma^2 = 5/8\, \sigma^2$ 1

$V(\theta_2) > V(\theta_1)$ alors on choisit $\theta_1$. 1